## 5. Parameter sensitivity analysis by using thermoporoelastic framework

Where:

:volumetric strain, is equivalent to

:variation in fluid content, positive for fluid entering the frame

:entropy density

:biot effective stress coefficient

:drained thermoelastic effective stress coefficient, which plays a role similar to in defining the effective stress relation

: Skempton pore pressure coefficient

:total compressive stress, is equivalent to /3

:pore pressure

:incremental temperature

:drained bulk modulus

: shear modulus

: Poisson’s ratio

:skempton pore pressure coefficient

:coefficient of volumetric thermal expansion of solid

:coefficient of volumetric thermal expansion of porosity

:coefficient of volumetric thermal expansion of fluid

:coefficient of volumetric thermal expansion for variation in fluid content, also define as differential expansion coefficient between fluid and solid

:drained coefficient of volumetric thermal expansion of porous medium frame



:constitutive constant, is the ratio of specific heat of the porous medium (solid and fluid) at constant stress over the reference absolute temperature,

:fluid flux

:heat flux

:absolute temperature

:permeability coefficient

:effective thermal conductivity

:mechano-caloric coefficient

:thermo-osmosis coefficient







 where 

Where





For the fluid mass balance equation:



And thermal energy balance equation：



Hydraulic transport law：



Thermal transport law：



By combining the balance equation and the transport laws, the pore pressure diffusion and thermal diffusion equations could be obtained as follow:





Inserting the constitutive equations into the diffusion equations,

We could obtain the





By taking the irrotational field assumption, the integral of Naiver equation then becomes following equations.





Then the couple diffusion equations becomes:





By rearranging the equations above, we have the coupled diffusion equations as following:

In other form as



Where













Where

, and

The two above differential equations are coupled in terms of T and p, which can be transformed into two uncoupled diffusion equations in terms of  and by using the approach developed by Sarout and Detournay (Ref). Their approach is starting with obtaining two eigenvalues  and  of matrix. Then the eigen-decomposition theorem (ref. Bunger.ref.23) will allow us to define a transition matrix which is composed of eigenvalues and eigenvectors of . So temperature and pore pressure can be expressed as follows



Where





Then the eigen-decomposition theorem proves that this transition leads directly to an uncoupled system of diffusion equations for the given by

 where 

Then by applying the Laplace transform to the eigen function , i.e., in equation (53), the equation (52) becomes an ordinary differential equation in terms of variable , where the is function of the coordinators in cylindrical system ρ, and the Laplace parameter s and the ’s eigenvalue (as shown in equation 54)

 where 

Therefore, the coupled partial differential diffusion equation (50) has been lead to the zeroth-order modified Bessel equation which is shown in (58) and with the general solution as (59) (ref. Bunger).  and  are unknowns that will be determined based on the boundary conditions, and  and are the zeroth-order modified Bessel functions of the first and second kinds, respectively.



In the P&A applications, the length of the plug is usually more longer than its diameter, so it is reasonable to simplify this problem into axial symmetry and plane strain problem. The symmetry conditions given by the cement plug will ensure the equals to zero.





This solution is numerically inverted to the time domain using Stehfest’s method, which has been proved to be efficient in poroelastic problems. For a given function p with Laplace transform , Stehfest’s method can be expressed as, taking pore pressure for example:



Where the coefficient are given by



2. The derivations of the radial stress and radial displacement

By integrating the results from irrotational field, we could obtain the radial displacement by integrating it once and obtain the displacement as follows



Based on the strain-stress relationship under the plain-strain conditions, we could arrive the following:

 where  and 

Thus, three unknowns can be determined from the stress boundary conditions:



Thus, three unknowns can be determined from the displacement boundary conditions:













Where



















